The nucleon mass and sigma term from lattice QCD

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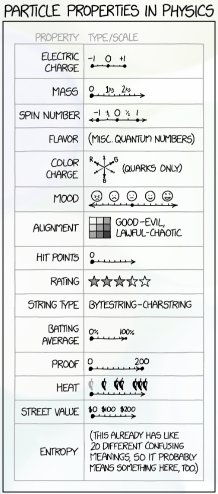
Really, the nucleon mass?

The nucleon mass is one of the most precisely measured quantities in physics

- Experiment: relative uncertainty to ~ 1 part per 10 billion
- Lattice: relative uncertainty to ~ 1 part per 100

With the lattice, we can...

- ...check theory against experiment
- ...study convergence of heavy baryon χPT
- ...use M_N to access other observables (eg, $\sigma_{N\pi}$)



xkcd:1862

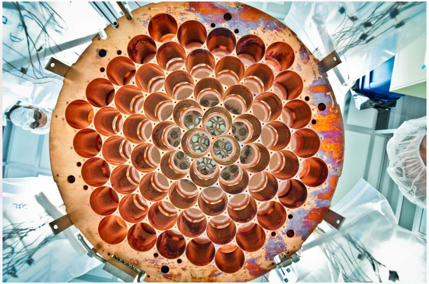
The sigma terms: what are they and what are they good for?

By definition, the sigma terms are the quark condensates inside the nucleon

$$\sigma_q = m_q \langle N|\overline{q}q|N\rangle$$

These parameterize:

- the q-quark mass shift to M_N
- the coupling to the Higgs
- the spin-independent coupling to some dark matter candidates



Large Underground Xenon experiment

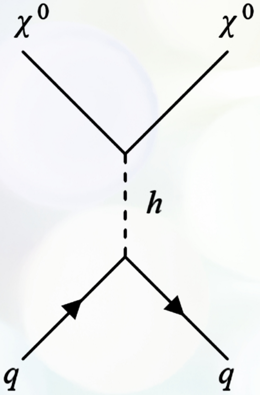
Phenomenological significance of the nucleon-pion sigma term

Let χ be the lightest neutralino from the minimal supersymmetric extension to the Standard Model (MSSM).

$$\mathcal{L}_q = \sum_i \underbrace{\alpha_{3i}\,\overline{\chi}\chi\,\overline{q}_i q_i}_{\text{spin-independent}} + \sum_i \underbrace{\alpha_{2i}\,\overline{\chi}\gamma_\mu\gamma_5\chi\,\overline{q}_i\gamma^\mu\gamma_5 q_i}_{\text{spin-dependent}}$$

MSSM direct dark matter experiments look for scattering off nuclei

- interactions either spin-independent (σ_q) or spin-dependent (g_A^q)
- spin-dependent cross section suppressed by $\beta^2 = (v/c)^2$



[Thornberry; doi:10.1140/epjs/s11734-021-00093-1]

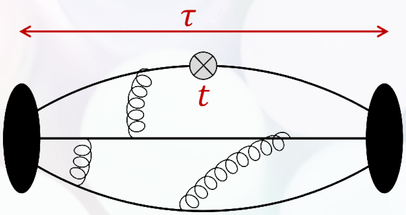
Two paths to the sigma term

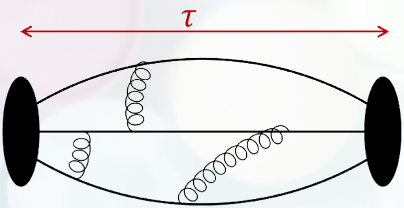
The direct approach:

- Generate $\sigma_{N\pi} = \hat{m}\,\langle N|\overline{u}u + \overline{d}d|N\rangle$ per lattice ensemble
- Fit the 3-point function
- Extrapolate $\sigma_{N\pi}$ to the physical point

The Feynman-Hellman approach:

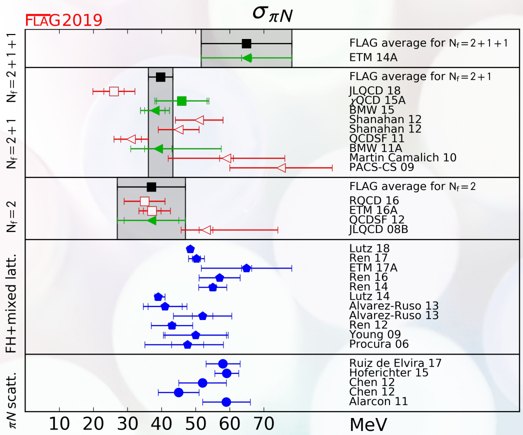
- Generate $C(t) = \langle 0|O_N^\dagger(t)O_N(0)|0\rangle$
- Fit the 2-point function
- Extrapolate M_N to the physical point
- Calculate $\sigma_{N\pi} = \hat{m}\frac{\partial M_N}{\partial \hat{m}}|_{\text{phys point}}$



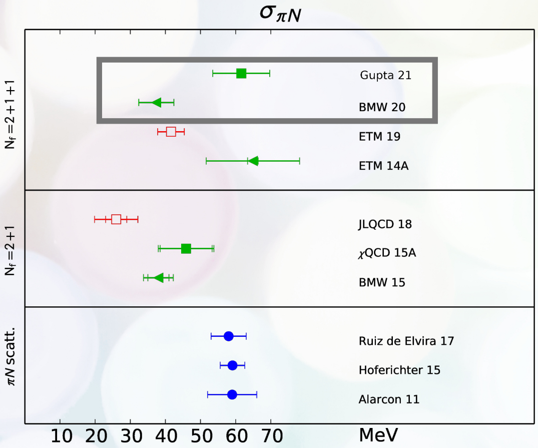


[FLAG; arXiv:1902.08191]

Previous work



[FLAG, 2019; arXiv:1902.08191]



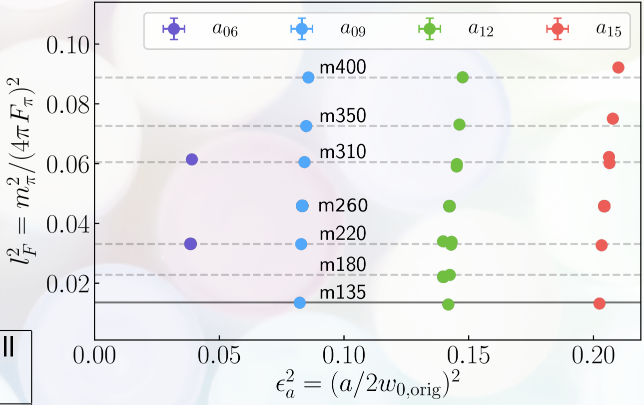
[Gupta, 2021; arXiv:2105.12095]

Project objectives & lattice details

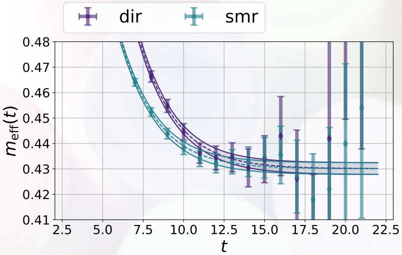
Objectives:

1. Fit correlators
2. Extrapolate masses to the phys point
3. Calculate $\sigma_{N\pi}$ via the Feynman-Hellman theorem

Action	Valence: Domain-wall Sea: staggered
Gauge configs	MILC – thanks!
m_π	130 - 400 MeV
a	0.06 - 0.15 fm
Scale setting?	Done!

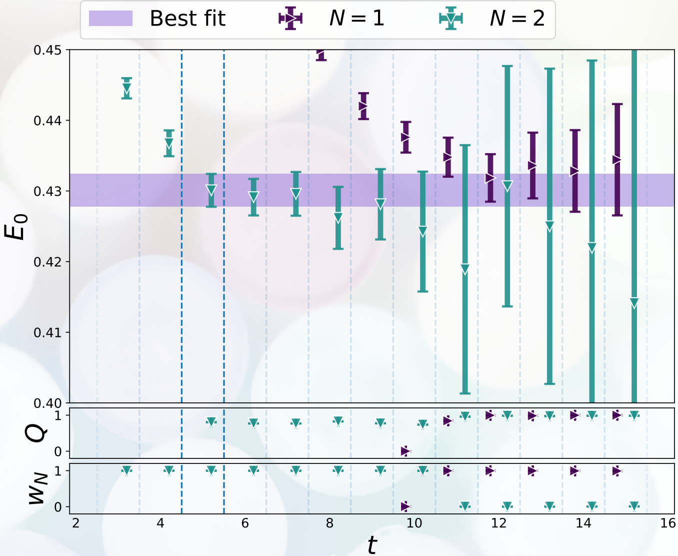


N correlator fits (a09m135)



$$C(t) = \sum_n A_n e^{-E_n t}$$

$$\implies m_{\text{eff}}(t) = \log \frac{C(t)}{C(t+1)}$$



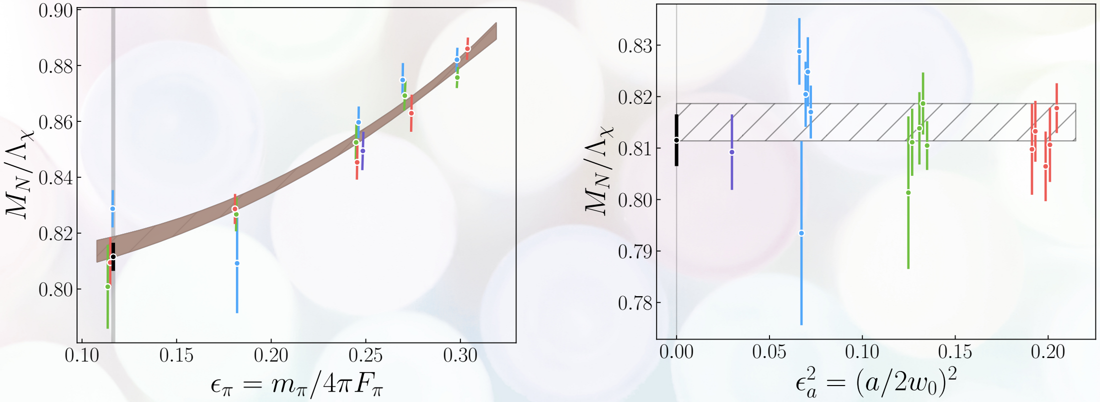
Fit strategy: mass formulae

Instead of fitting M_N, fit dimensionless M_N/Λ_χ $\qquad (\Lambda_\chi = 4\pi F_\pi\,,\ \epsilon_\pi = m_\pi/\Lambda_\chi)$

$$
\begin{aligned}
\frac{M_N}{4\pi F_\pi} = \ & c_0 && \text{(LLO)}\\
& + \left(\beta_N^{(2)} - c_0\bar{\ell}_4^r\right)\epsilon_\pi^2 + c_0\epsilon_\pi^2 \log \epsilon_\pi^2 && \text{(LO)}\\
& - \frac{3\pi}{2} g_{\pi NN}^2 \epsilon_\pi^3 && \text{(NLO)}\\
& + \left(\beta_N^{(4)} + c_0\left(\bar{\ell}_4^r\right)^2 - c_0\beta_F^{(4)}\right)\epsilon_\pi^4 && \text{(N}^2\text{LO)}\\
& \quad - \frac{1}{4} c_0\epsilon_\pi^4 \left(\log \epsilon^2\right)^2 + \left(\alpha_N^{(4)} - c_0\alpha_F^{(4)} - 2c_0\bar{\ell}_4^r\right)\epsilon_\pi^4 \log \epsilon_\pi^2
\end{aligned}
$$

- The $1/4\pi F_\pi$ expansion doesn't *require* fitting additional LECs; it only adds some log terms
- We'd like to push this M_N/Λ_χ analysis as far as possible

M_N/Λ_χ extrapolations



- Fit has negligible lattice spacing dependence

Expansion of $\sigma_{N\pi}$

Expand $\sigma_{N\pi} = \hat{m}\frac{\partial M_N}{\partial \hat{m}} \to \hat{m}\frac{\partial}{\partial \hat{m}} = \frac{1}{2}\epsilon_\pi(\cdots)\frac{\partial}{\partial \epsilon_\pi}$

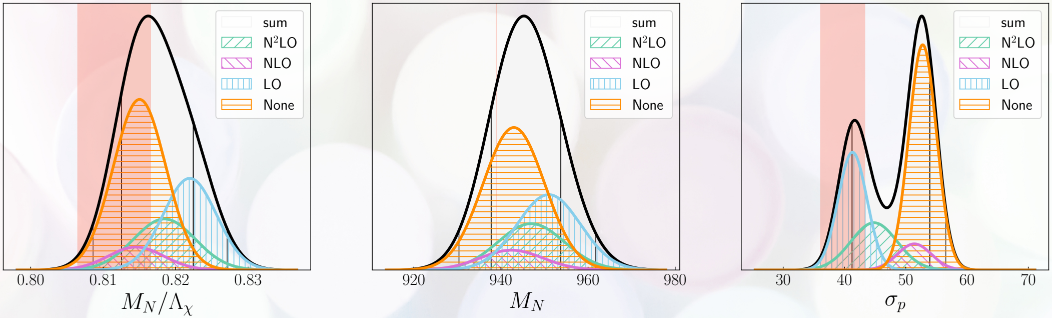
$$\sigma_{N\pi} = \frac{1}{2}\epsilon_\pi\left[1 + \epsilon_\pi^2\left(\frac{5}{2} - \frac{1}{2}\bar{\ell}_3 - 2\bar{\ell}_4\right) + \mathcal{O}\left(\epsilon_\pi^3\right)\right]\overbrace{\left[\Lambda_\chi^*\frac{\partial\left(M_N/\Lambda_\chi\right)}{\partial \epsilon_\pi} + \frac{M_N^*}{\Lambda_\chi^*}\frac{\partial \Lambda_\chi}{\partial \epsilon_\pi}\right]}^{\frac{\partial M_N}{\partial \epsilon_\pi}}$$

$$\frac{1}{2}\epsilon_\pi\Lambda_\chi^*\frac{\partial\left(M_N/\Lambda_\chi\right)}{\partial \epsilon_\pi} = \frac{1}{2}\Lambda_\chi^*\left[\left(-2c_0\left(\bar{\ell}_4 - 1\right) + 2\beta_N^{(2)}\right)\epsilon_\pi^2 + \mathcal{O}\left(\epsilon_\pi^3\right)\right] \sim 10 \text{ MeV}$$

$$\frac{1}{2}\epsilon_\pi\frac{M_N^*}{\Lambda_\chi^*}\frac{\partial \Lambda_\chi}{\partial \epsilon_\pi} = \frac{1}{2}M_N^*\left[2\left(\bar{\ell}_4 - 1\right)\epsilon_\pi^2 + \mathcal{O}\left(\epsilon_\pi^3\right)\right] \sim 40 \text{ MeV}$$

- Fitting M_N/Λ_χ requires an extra term
- Largest contribution comes from second term $\implies$ $\bar{\ell}_4$ must be precisely determined

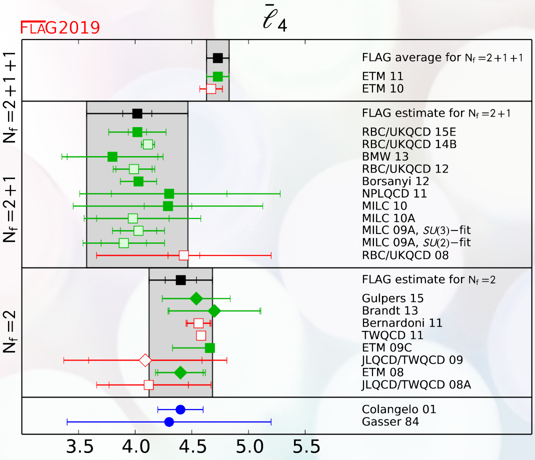
Comparing χPT terms by order



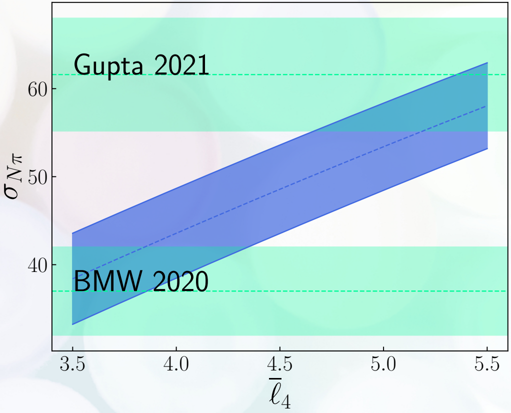
Here we use:

- F_π-derived χPT terms up to $\mathcal{O}(\epsilon_\pi^2)$ & M_N-derived χPT terms up to $\mathcal{O}(\epsilon_\pi^4)$
- FLAG average for $\bar{\ell}_4$

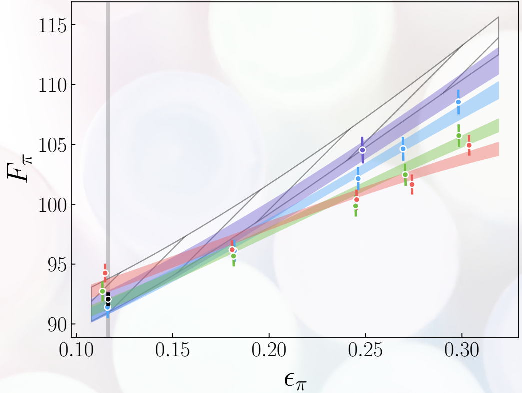
$\sigma_{N\pi}$ as a function of $\bar{\ell}_4$

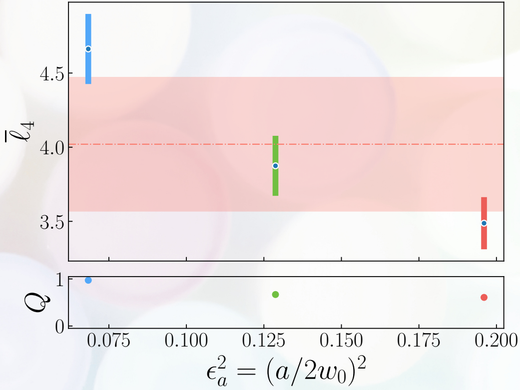


[FLAG, 2019; arXiv:1902.08191]



F_π extrapolation: $\mathcal{O}(\epsilon_\pi^2)$ χPT + $\mathcal{O}(a^4)$





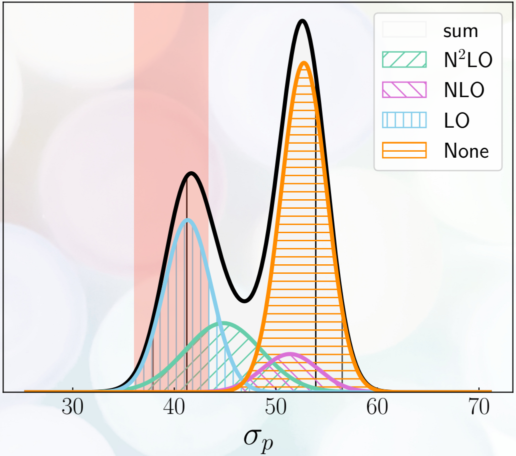
$$F_\pi = F_0 \left[1 + \epsilon_\pi^2 \left(\bar{\ell}_4^r \left(\mu = 4\pi F_\pi \right) - \log \epsilon_\pi^2 \right) + \mathcal{O}\left(\epsilon_\pi^4 \right) \right]$$

$$\bar{\ell}_4^r(\mu) = (4\pi)^2 \ell_4^r(\mu) \qquad \bar{\ell}_4 = \bar{\ell}_4^r(\mu) - \log \left[\frac{(m_\pi^*)^2}{\mu^2} \right]$$

Summary & future work

In conclusion:

- Tension exists between phenomenology and the lattice w.r.t. $\sigma_{N\pi}$
- Can extract $\sigma_{N\pi}$ from a dimensionless fit of M_N/Λ_χ
- However, this requires a precise determination of the LECs associated with the chiral expression for F_π



To do:

- Carefully determine F_π LECs for $\sigma_{N\pi}$
- Add FV corrections

$$
\begin{aligned}
\frac{M_N}{4\pi F_\pi} = \; & c_0 && \text{(LLO)} \\
& + \left(\beta_N^{(2)} - c_0 \overline{\ell}_4^r\right) \epsilon_\pi^2 && \text{(LO)} \\
& - \frac{3\pi}{2} g_{\pi NN}^2 \epsilon_\pi^3 - \frac{4}{3} g_{\pi N\Delta}^2 \mathcal{F}(\epsilon_\pi, \epsilon_{N\Delta}, \mu) && \text{(NLO)} \\
& + \gamma_N^{(4)} \epsilon_\pi^2 \mathcal{J}(\epsilon_\pi, \epsilon_{N\Delta}, \mu) - \frac{1}{4} c_0 \epsilon_\pi^4 \left(\log \epsilon^2\right)^2 && \text{(N}^2\text{LO)} \\
& \quad + \left(\alpha_N^{(4)} - c_0 \alpha_F^{(4)} - 2 c_0 \overline{\ell}_4^r\right) \epsilon_\pi^4 \log \epsilon_\pi^2 \\
& \quad + \left(\beta_p^{(4)} + c_0 \left(\overline{\ell}_4^r\right)^2 - c_0 \beta_F^{(4)}\right) \epsilon_\pi^4
\end{aligned}
$$

Some observations:

- The $1/4\pi F_\pi$ expansion doesn't *require* fitting additional LECs
- The Δ-χPT terms push the fit downwards