V_{us} from hyperon semileptonic decays

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Hyperonic observables on the lattice

Soon LHCb will have millions of hyperon semileptonic decays available for analysis. We propose to calculate transition form factors which, when combined with measurements of decay widths from LHCb, will be used to determine the Cabibbo–Kobayashi–Maskawa (CKM) matrix element V_{us}. Along the way, we will also calculate the hyperon mass spectrum and axial charges as a test of baryon chiral perturbation theory, which will serve as the framework for the form factor calculations.

V_{us} as a test of the Standard Model

Since the '50s, physicists have known that strangess is not conserved by the weak interaction. In fact, because the quark eigenstates of the strong and weak interaction are different, no quark flavor is conserved by the weak interaction. This mismatch is encoded in the CKM matrix.

$$\begin{bmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{bmatrix} = \begin{bmatrix} 0.97446(10) & 0.22452(44) & 0.00365(12) \\ 0.22438(44) & 0.97359(11) & 0.04214(76) \\ 0.00896(24) & 0.04133(74) & 0.99910(03) \end{bmatrix}$$

The Standard Model predicts that the CKM matrix is unitary. From this requirement follows the "top-row unitarity" condition.

$$\underbrace{|V_{ud}|^2}_{\text{Known from experiments}} + \underbrace{|V_{us}|^2}_{\text{Accessible by lattice}} + \underbrace{|V_{ub}|^2}_{\text{Relatively small}} = 1$$

Of these three matrix elements, V_{ud} can be measured the most precisely while V_{ub} is almost negligible. Although V_{us} can be determined purely experimentally, the most precise determinations of V_{us} require lattice QCD calculations of the form factors.

Techniques for determining V_{us}

V_{us} is determined using one of four sources:

- Leptonic K decays (previous work of ours)
- Semi-leptonic K decays
- Hyperonic decays (goal of this project)
- Tau hadronic decays

The most straightforward calculation comes from leptonic decays, which requires us to calculate only a single form factor.

$$K^- \{ \cdots = \frac{G_F}{\sqrt{2}} V_{ud} \underbrace{\langle 0 | \bar{s}\gamma_\mu\gamma_5 u | K(p)\rangle}_{ip_\mu F_K} \mu \gamma^\mu (1-\gamma^5)\bar{\nu}_\mu$$

Through Fermi's golden rule, we can relate this transition matrix element to the decay widths.

$$d\Gamma(K \to \mu\bar{\nu}_\mu) \propto |T_{fi}|^2 d\Phi \quad \Longrightarrow \quad \Gamma(\pi \to \mu\bar{\nu}_\mu) \propto |V_{us}|^2 F_K^2$$

Determining V_{us} from hyperon decays follows a similar procedure; however, the process is slightly by complicated by the need for multiple form factors. In particular, although the purely leptonic decay transition element only has a vector part, the hyperon matrix element can be split into a vector and axial part.

Unlike the vector form factor from leptonic decay, the axial form factor from hyperon decay is not protected by the Ademello-Gatto theorem. Therefore SU(3) breaking effects are expected to play a larger role when extracting V_{us} from hyperon decays, necessitating a study of baryon chiral pertubation theory.

V_{us} from F_K/F_π

As a variation of the pure leptonic decay calculation, Marciano has shown how we can relate the ratio of the kaon and pion decay constants F_K/F_π to the ratio V_{us}/V_{ud}. This provides an independent and competitve determination of V_{us}.

$$\frac{\Gamma(K \to l\bar{\nu}_l)}{\Gamma(\pi \to l\bar{\nu}_l)} = \left(\frac{F_K}{F_\pi}\right)^2 \frac{|V_{us}|^2}{|V_{ud}|^2} \frac{m_K(1-m_l^2/m_K^2)^2}{m_\pi(1-m_l^2/m_\pi^2)^2} \Big[1 + \overbrace{\delta_{\text{EM}} + \delta_{\text{SU(2)}}}^{\text{QED/isospin}}\Big]$$

In this vein, we can determine V_{us} by calculating F_K/F_π on the lattice. Marciano's formulation has several advantages:

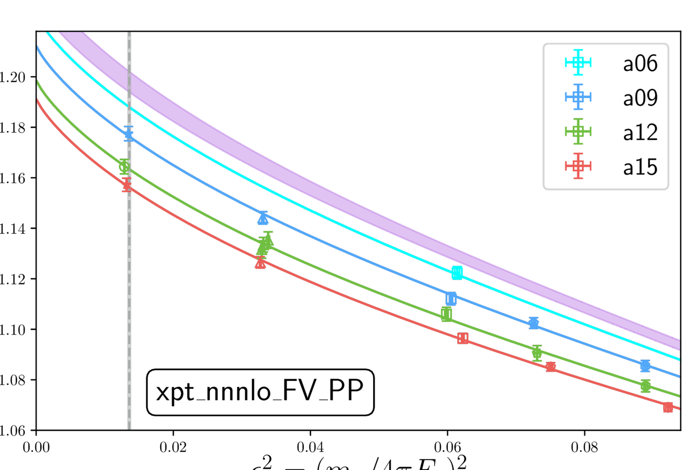
- F_K/F_π is dimensionless ⇒ scale setting is unnecessary
- F_K, F_π correlated ⇒ increased precision
- Mesonic, not baryonic ⇒ no signal-to-noise problems from baryonic operators
- Chiral extrapolation known to $O(m_\pi^2)$ (NNLO) ⇒ limited by statistics

With F_K, F_π generated on the lattice, we can construct an expression for the observable F_K/F_π using chiral perturbation theory. By determining the low energy constants (LECs) of the chiral expression, we can extrapolate our lattice data to the physical point.

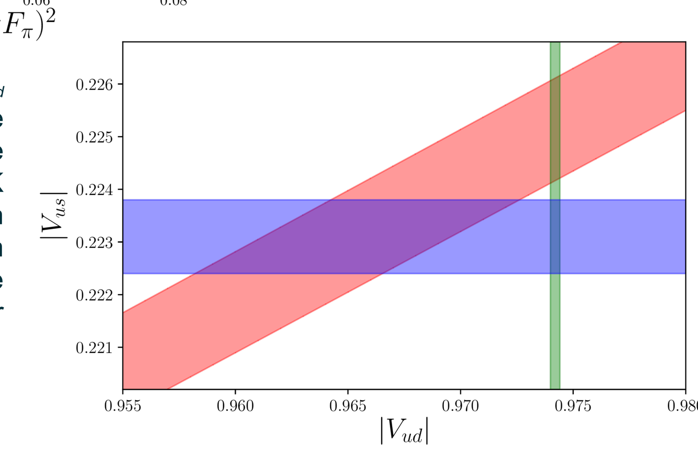
Defining $\epsilon_p = m_p/\Lambda_\chi$, the chiral expression to NLO is

$$\begin{aligned} \left(\frac{F_K}{F_\pi}\right)_{\chi\text{PT}} \approx\ & 1 && \text{(LO)} \\ & + \frac{5}{8}\epsilon_\pi^2 \log\epsilon_\pi^2 - \frac{1}{4}\epsilon_K^2\log\epsilon_K^2 - \frac{3}{8}\epsilon_\eta^2\log\epsilon_\eta^2 && \text{(NLO)} \\ & + 4(4\pi)^2 \underbrace{(\epsilon_K^2 - \epsilon_\pi^2)}_{\text{SU(3) flavor}} \underbrace{L_5}_{\text{LEC}} \end{aligned}$$

In our work, we considered chiral terms up to NNLO (written out, the expression would be too large to fit on this poster). We considered several trucations of the chiral terms (including a comparison to a pure Taylor expansion), as well as multiple parameterizations of the chiral cutoff and lattice artifacts. In total, we examined 216 different models; however, as most of these models had negligible weight, we further limited our study to 24 models when performing the model average.



Left: an example fit. The different bands display extrapolations at different lattice spacings, with the purple band showing the extrapolation at the continuum point. The intersection of the vertical line and the purple band is the extrapolation of F_K/F_π to the physical point.



Right: our determination of V_{us}/V_{ud} (red band). The horizontal blue band denotes the global lattice average of V_{us} from semileptonic K decays, while the vertical green band denotes the V_{ud} average from the PDG. The intersection of the green and red bands is our determination of V_{us}.

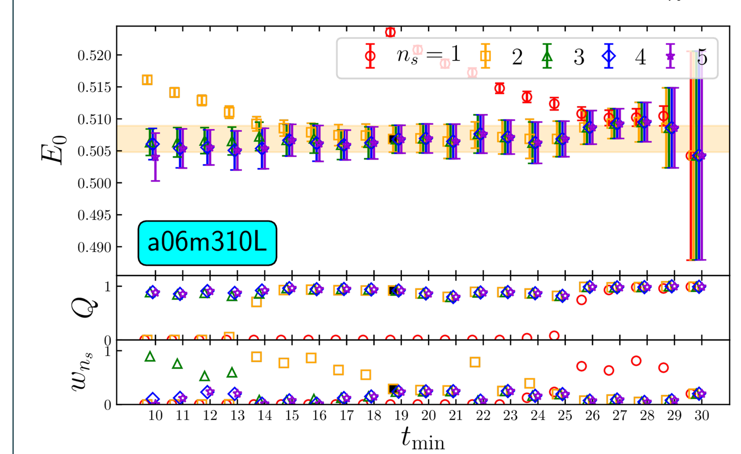
$$F_K^\pm/F_\pi^\pm = 1.1964(44) \quad \Longrightarrow \quad \sum_{q\in\{d,s,b\}} |V_{uq}|^2 = 0.99977(59)$$

Scale setting with w_0

In order to convert the dimensionless lattice observables into physical units, we performed a procedure known as scale setting. In our work, we used the Ω baryon mass to set the scale.

We determine the Ω mass in lattice units on some lattice (ensemble) through the two-point function.

$$C(t) = \langle 0|O(t)O^\dagger(0)|0\rangle \approx \sum_n A_n e^{-E_n t}$$



Left: stability plot for the Ω on some ensemble. Here we consider fits fixing t_{max} while varying t_{min} and the number of states n_s. Since early times are contaminated by excited state contributions and late times have low signal-to-noise, the "best" fit involves only a small window of times.

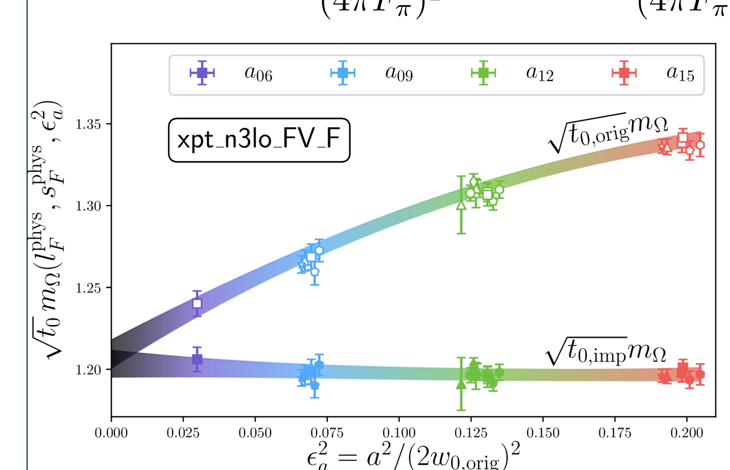
On this particular ensemble, the lattice spacing is $a \sim 0.06$ fm. Thus

$$(aE_0)^{\text{latt}} = 0.5069(21) \quad \Longrightarrow \quad E_0^{\text{phys}} = \frac{(aE_0)^{\text{latt}}}{a} \sim 1700 \text{ MeV}$$

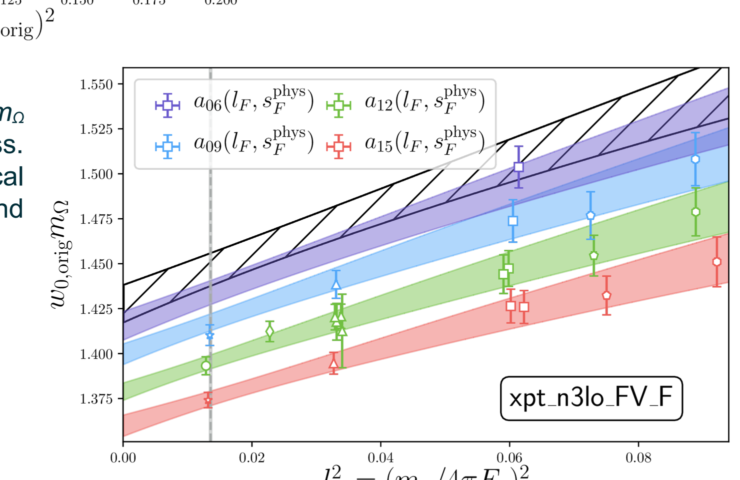
which is near the physical point value of 1672.45(29) MeV, albeit with unquantified uncertainties from systematics.

Given the values of the $am_\Omega(m_\pi, m_K, a)$ on many ensembles, we extrapolate to the physical point $m_\Omega(m_\pi=m_\pi^{\text{expt}}, m_K=m_K^{\text{expt}}, a=0)$. Comparing the extrapolated value with the experimental value allows us to set the scale. We perform the extrapolation using these dimensionless quantities:

$$l_F^2 = \frac{m_\pi^2}{(4\pi F_\pi)^2} \qquad s_F^2 = \frac{2m_K^2 - m_\pi^2}{(4\pi F_\pi)^2} \qquad \epsilon_a^2 = \frac{a^2}{4w_0^2}$$



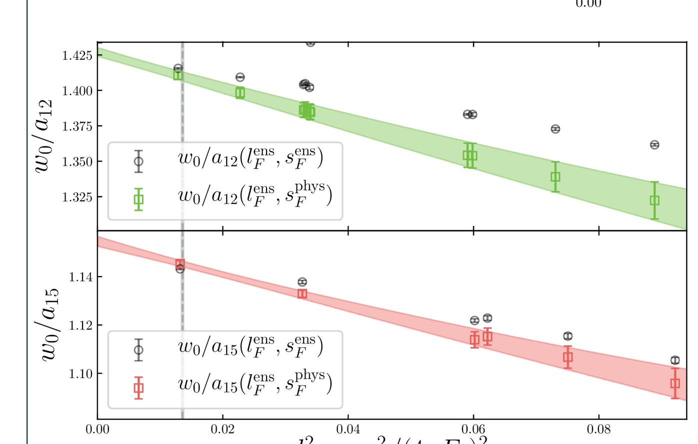
Right: chiral extrapolation of $m_\Omega\sqrt{t_0}$ as a function of lattice spacing. Depending on the observable used for scale setting, the observable can approach the physical point quite differently.



Right: chiral extrapolation of $w_0 m_\Omega$ as a function of light quark mass. By extrapolating to the physical point (starred quantities), we find the physical value of w_0 to be

$$w_0^* = \frac{(w_0 m_\Omega)^*}{m_\Omega^{\text{expt}}}$$

$$w_0^* = 0.1709(11)$$



Left: chiral interpolation of w_0/a at different lattice spacings. By using the interpolated value of w_0/a at a given lattice spacing in conjunction with the extrapolated value of w_0, we can determine the physical value of an observable on a given lattice. For the Ω example above:

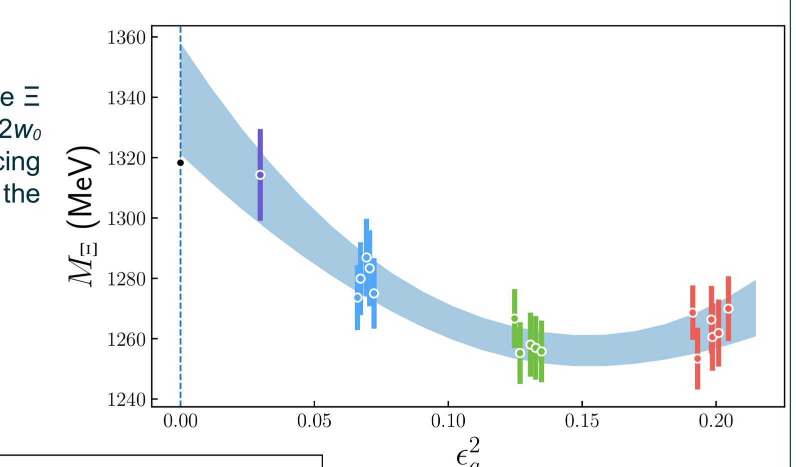
$$E_0^{\text{phys}} = \frac{w_0/a_{09}}{w_0^*}(a_{09}E_0)^{\text{latt}} = 1749(16) \text{ MeV}$$

Hyperon masses and axial charges

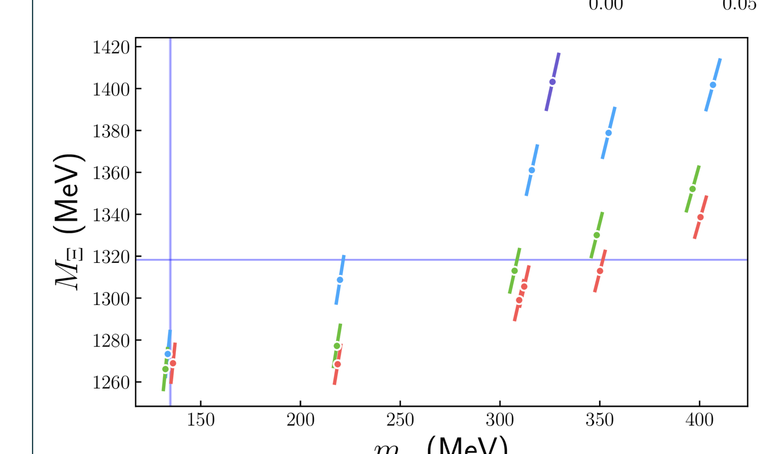
A hyperon is a baryon containing at least one strange quark and no heavier quarks. Besides their role in extracting V_{us}, hyperons are thought to potentially play an important role in neutron stars. Although hyperons quickly decay in the lab, under the immense pressure of a neutron star, hyperons could potentially be stable over timespans of millions of years. Understanding hyperon properties such as their masses and axial charges are therefore integral when modeling the equation of state of a neutron star.

Employing SU(2) chiral perturbation theory, we are able to construct mass formulae for the 6 hyperons of the baryon decuplet. As an example, the chiral mass formula for the cascade is given below.

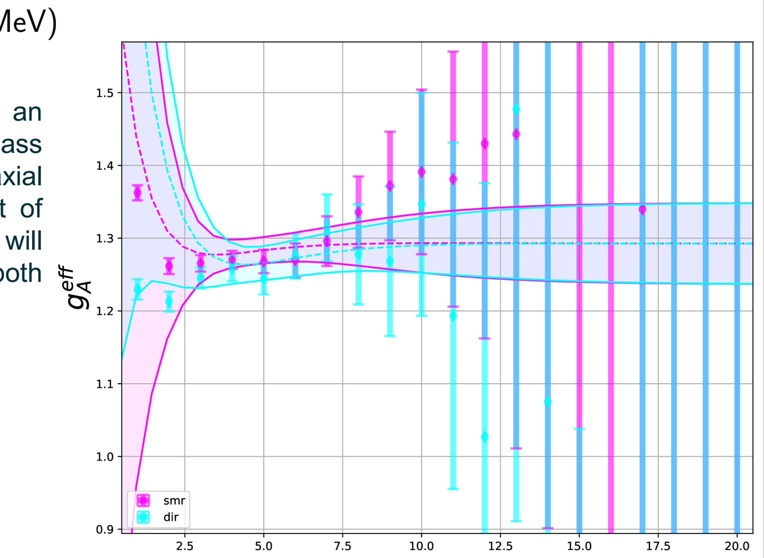
$$\begin{aligned} M_\Xi^{(\chi)} =\ & M_\Xi^{(0)} && \text{(LLO)} \\ & + \sigma_\Xi \Lambda_\chi \epsilon_\pi^2 && \text{(LO)} \\ & - \frac{3\pi}{2} g_{\pi\Xi\Xi}^2 \Lambda_\chi \epsilon_\pi^3 - g_{\pi\Xi^*\Xi}^2 \Lambda_\chi \mathcal{F}(\epsilon_\pi, \epsilon_{\Xi\Xi^*}, \mu) && \text{(NLO)} \\ & + \frac{3}{2} g_{\pi\Xi^*\Xi}^2 (\sigma_\Xi - \bar{\sigma}_\Xi)\Lambda_\chi \epsilon_\pi^2 \mathcal{J}(\epsilon_\pi, \epsilon_{\Xi\Xi^*}, \mu) && \text{(N}^2\text{LO)} \\ & + \alpha_\Xi^{(4)} \Lambda_\chi \epsilon_\pi^4 \log\epsilon_\pi^2 + \beta_\Xi^{(4)} \Lambda_\chi \epsilon_\pi^4 \end{aligned}$$



Right: a representative fit of the Ξ mass. Here $\epsilon_a = a / 2w_0$ characterizes the lattice spacing dependence. The black dot is the PDG value.



Right: mass of Ξ on various lattices, sorted by pion mass. Vertical/horizontal bands denote physical values.



Right: An axial charge fit on an ensemble. Notice that chiral mass formula depends on the axial charges g. A simultaneous fit of both observables, therefore, will lead to improved precision in both extrapolations.

Publications

- Miller, Nolan et al. "Scale setting the Möbius domain wall fermion on gradient-flowed HISQ action using the omega baryon mass and the gradient-flow scales t_0 and w_0". Physical Review D 103. 5(2021).
- Miller, Nolan et al. "F_K/F_π from Möbius domain-wall fermions solved on gradient-flowed HISQ ensembles". Physical Review D 102. 3(2020).

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