On hyperon masses, axial charges, and form factors

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Why hyperons?

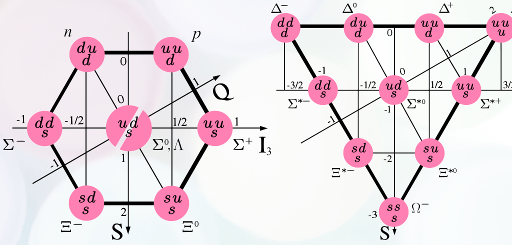
Hyperon: a baryon containing at least one strange quark but no heavier quarks

Why study hyperons?

- Decays $\implies V_{us} \implies$ top-row unitarity: $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 \stackrel{?}{=} 1$
- Axial charge, mass spectra important for neutron star equation of state
- Test heavy baryon χPT

Why the lattice?

- Hypernuclear structure harder to study experimentally
- Hyperons decay rapidly in the lab ($\tau < 1$ ns)

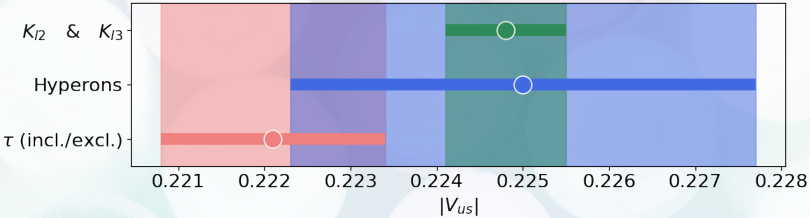


[Wikipedia]

Experimental determination of $|V_{us}|$?

Experimental results are less precise without lattice QCD input

- Leptonic/semi-leptonic K decays: requires LQCD estimate of F_K/F_π or $f^+(0)$
- Hyperon decays: requires estimate of axial charge, vector charge, and other form factors; new results from LHCb could make this competitive
- τ hadronic decays (eg, $\tau^- \to \pi^- \nu_\tau$): LQCD not required, but there are theory problems



Tension between K_{l2} & K_{l3}

K_{l2} ($f_{K^\pm}/f_{\pi^\pm}$):

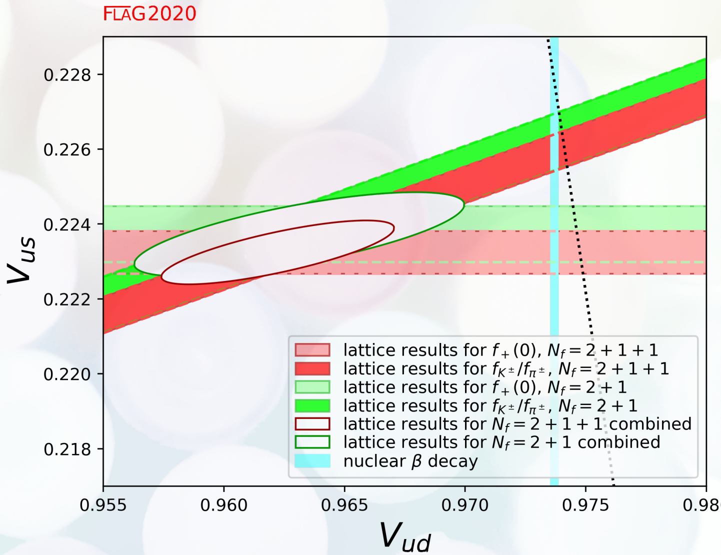
$$\sum_{q\in\{d,s,b\}} |V_{uq}|^2 = 0.99883(37)$$

$\Longrightarrow$ 3.2 σ deviation

K_{l3} ($f_+(0)$):

$$\sum_{q\in\{d,s,b\}} |V_{uq}|^2 = 0.99794(37)$$

$\Longrightarrow$ 5.6 σ deviation



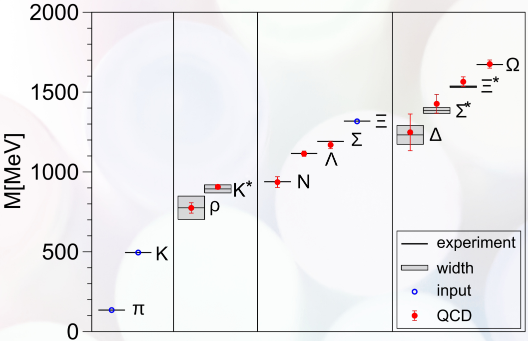
Project goals & lattice details

Project Goals:

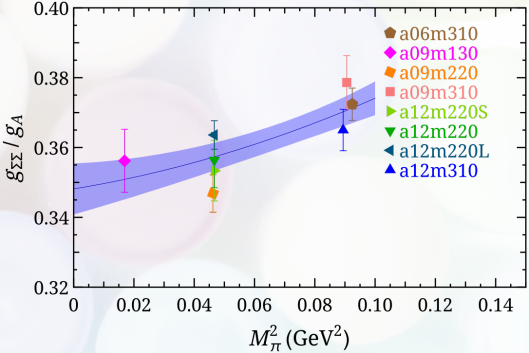
1. Determine the hyperon mass spectrum
2. Determine axial/vector charges
3. Test convergence of SU(2) HBXPT for hyperons
4. Calculate hyperon-to-nucleon form factors

Action	Valence: Domain-wall Sea: staggered
m_π	130 - 400 MeV
a	0.06 - 0.15 fm
Scale setting?	Done!

Previous work

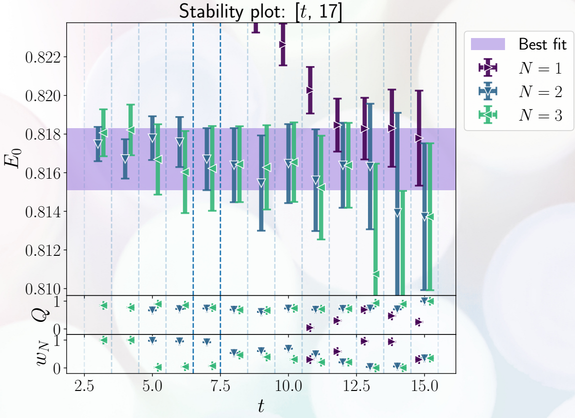


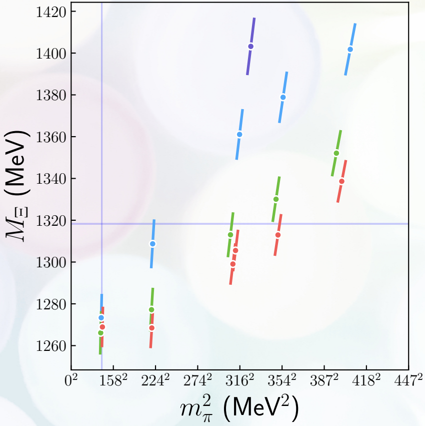
[BMW, 2009; arXiv:0906.3599]



[Savanur & Lin, 2018; arXiv:1901.00018]

Ξ correlator fits





Fit strategy: mass formulae

Consider the $S = 2$ hyperons in the isospin limit

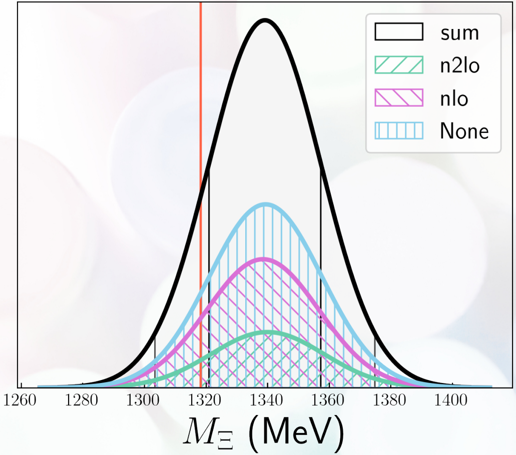
$$
\begin{aligned}
M_{\Xi}^{(\chi)} = \; & M_{\Xi}^{(0)} \\
& + \sigma_{\Xi} \Lambda_\chi \epsilon_\pi^2 \\
& - \frac{3\pi}{2} g_{\pi\Xi\Xi}^2 \Lambda_\chi \epsilon_\pi^3 \\
& \quad - g_{\pi\Xi^*\Xi}^2 \Lambda_\chi \mathcal{F}(\epsilon_\pi, \epsilon_{\Xi\Xi^*}, \mu) \\
& + \frac{3}{2} g_{\pi\Xi^*\Xi}^2 (\sigma_{\Xi} - \overline{\sigma}_{\Xi}) \Lambda_\chi \epsilon_\pi^2 \mathcal{J}(\epsilon_\pi, \epsilon_{\Xi\Xi^*}, \mu) \\
& \quad + \alpha_{\Xi}^{(4)} \Lambda_\chi \epsilon_\pi^4 \log \epsilon_\pi^2 + \beta_{\Xi}^{(4)} \Lambda_\chi \epsilon_\pi^4
\end{aligned}
$$

$$
\begin{aligned}
M_{\Xi^*}^{(\chi)} = \; & M_{\Xi^*}^{(0)} \\
& + \overline{\sigma}_{\Xi} \Lambda_\chi \epsilon_\pi^2 \\
& - \frac{5\pi}{6} g_{\pi\Xi^*\Xi^*}^2 \Lambda_\chi \epsilon_\pi^3 \\
& \quad - \frac{1}{2} g_{\pi\Xi^*\Xi}^2 \Lambda_\chi \mathcal{F}(\epsilon_\pi, -\epsilon_{\Xi\Xi^*}, \mu) \\
& + \frac{3}{4} g_{\pi\Xi^*\Xi}^2 (\overline{\sigma}_{\Xi} - \sigma_{\Xi}) \Lambda_\chi \epsilon_\pi^2 \mathcal{J}(\epsilon_\pi, -\epsilon_{\Xi\Xi^*}, \mu) \\
& \quad + \alpha_{\Xi^*}^{(4)} \Lambda_\chi \epsilon_\pi^4 \log \epsilon_\pi^2 + \beta_{\Xi^*}^{(4)} \Lambda_\chi \epsilon_\pi^4
\end{aligned}
$$

Some observations:

- Many shared LECs between expressions $\implies$ fit simultaneously
- Mass fits depend on axial charges

Hyperon mass spectrum: Ξ preliminary results



$+1$:	Taylor $\mathcal{O}(m_\pi^2)$
$+1$:	χPT $\mathcal{O}(m_\pi^3)$
$+3$:	Taylor $\mathcal{O}(m_\pi^4) \oplus \chi$PT $\{0, \mathcal{O}(m_\pi^3), \mathcal{O}(m_\pi^4)\}$
5 :	chiral choices
$\times 5$:	chiral choices
$\times 2$:	$\{\mathcal{O}(a^2), \mathcal{O}(a^4)\}$
$\times 2$:	incl./excl. strange mistuning
$\times 2$:	Naïve priors or empirical priors
40 :	total choices

$$M_\Xi = 1339(17)^{s}(02)^{\chi}(05)^{a}(00)^{\text{phys}}(01)^{\text{M}}(??)^{\text{V}}$$

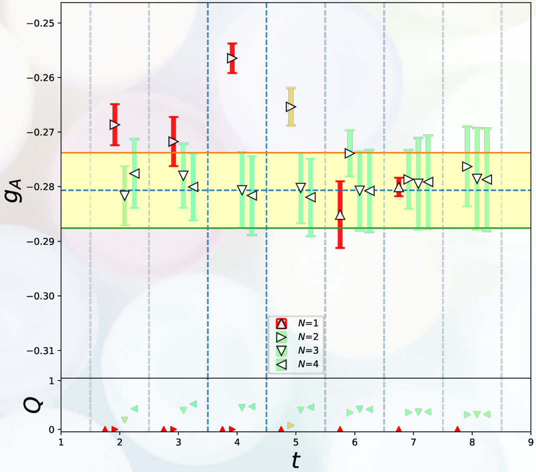
Summary & future work

In conclusion:

- Chiral mass and charge expressions share many LECs and would benefit from a simultaneous fit
- Hyperon decays provide an alternate method for extracting $|V_{us}|$
- Competitive if $O(1\%)$ determination of the form factors

To do:

- Add finite volume effects to mass fits
- (Simultaneously) fit axial charges
- Calculate hyperon-to-nucleon form factors



Transition matrix element for $B_1 \to B_2 + l^- + \overline{\nu}_l$

$$T_{\text{fi}} = \frac{G_F}{\sqrt{2}} V_{us} \left[\overbrace{\langle B_2 | \overline{u} \gamma_\mu \gamma^5 s | B_1 \rangle}^{\text{axial-vector}} - \overbrace{\langle B_2 | \overline{u} \gamma_\mu s | B_1 \rangle}^{\text{vector}} \right] \bar{l} \gamma^\mu (1 - \gamma^5) \nu_l$$

with hadronic matrix elements

$$\langle B_2 | \overline{u} \gamma_\mu \gamma^5 s | B_1 \rangle = g_A(q^2) \gamma_\mu \gamma^5 + \underbrace{\cancel{\frac{f_T(q^2)}{2M} i \sigma_{\mu\nu} q^\nu \gamma^5}}_{\text{G-parity}} + \frac{f_P(q^2)}{2M} q_\mu \gamma^5$$

$$\langle B_2 | \overline{u} \gamma_\mu s | B_1 \rangle = g_V(q^2) \gamma_\mu + \frac{f_M(q^2)}{2M} i \sigma_{\mu\nu} q^\nu + \overbrace{\cancel{\frac{f_S(q^2)}{2M}}}^{\text{CVC}} q_\mu$$

Let $M = \{\Pi, f\}$ denote a model. Per Bayes's theorem:

$$p(\Pi|D, f) = \frac{p(D|\Pi, f)p(\Pi|f)}{p(D|f)}$$

Assuming a uniform distribution for $p(\Pi|f)$:

$$\text{peak of } p(D|\Pi, f) \implies \text{peak of } p(\Pi|D, f)$$

where $p(D|\Pi, f)$ is the (readily available) likelihood

- Caveat: the uniformity assumption breaks down if we vary too many parameters separately or make our priors too narrow

Π	$p(D\|\Pi, f)$	$p(\Pi\|f)$
0 ± 0.1	0.27	0.33
0 ± 1	0.54	0.33
0 ± 10	0.19	0.33