Lattice calculation of F_K/F_π from a mixed domain-wall on HISQ action

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Why F_K/F_π?

The CabibboKobayashiMaskawa Matrix

Question: Are the quark eigenstates of the weak and strong interaction the same?

$$q_{\text{weak}} = V q_{\text{strong}}$$

Answer: No!

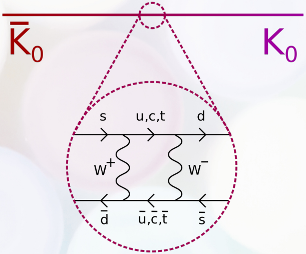


Figure: (Wikipedia)

$$\begin{bmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{bmatrix} = \begin{bmatrix} 0.97446(10) & 0.22452(44) & 0.00365(12) \\ 0.22438(44) & 0.97359(11) & 0.04214(76) \\ 0.00896(24) & 0.04133(74) & 0.999105(32) \end{bmatrix}$$

Unitarity of the CKM Matrix

Standard model predicts $V^\dagger V = 1$. In particular, we verify:

$$\underbrace{|V_{ud}|^2}_{\substack{\text{Known from}\\\text{experiments}}} + \underbrace{|V_{us}|^2}_{\substack{\text{Accessible}\\\text{by lattice}}} + \underbrace{|V_{ub}|^2}_{\substack{\text{Relatively}\\\text{small}}} = 1$$

Per Marciano:

$$\frac{\Gamma(K \to l\,\overline{\nu}_l)}{\Gamma(\pi \to l\,\overline{\nu}_l)} = \left(\frac{F_K}{F_\pi}\right)^2 \frac{|V_{us}|^2}{|V_{ud}|^2} \frac{m_K(1 - m_l^2/m_K^2)^2}{m_\pi(1 - m_l^2/m_\pi^2)^2} \left[1 + \frac{\alpha}{\pi}(C_K - C_\pi)\right]$$

$$\langle 0|\overline{d}\gamma_\mu\gamma_5 u|\pi^+(p)\rangle = i p_\mu F_\pi^+ \qquad \langle 0|\overline{s}\gamma_\mu\gamma_5 u|K^+(p)\rangle = i p_\mu F_K^+$$

Why *Lattice* QCD?

$|V_{us}|$ "easily" accessed by lattice, not experiment

- probe $|V_{us}|$ via $K \to l\nu, \pi l\nu$
- kaons decay rapidly

Lattice QCD is a non-pertubative approach to QCD

$$\langle O \rangle = \frac{1}{Z_0} \int \mathcal{D}[q, \overline{q}]\mathcal{D}[A]\, e^{iS_{\mathrm{QCD}}[q,\overline{q},A]} O[q, \overline{q}, A]$$

Lattice QCD pracitioners can:

- experiment with different discretizations of the QCD action
- control systematics (finite volume, lattice spacing, etc)
- fine-tune the QCD parameters to values different from those in our universe
- use lattice methods in tandem with EFT

Why F_K/F_π via Lattice QCD?

F_K/F_π is a *gold-plated* quantity, serving as an important benchmark for testing lattice QCD actions

- dimensionless $\implies$ scale-setting unnecessary
- F_K, F_π correlated $\implies$ high (sub-percent) precision
- mesonic, not baryonic $\implies$ no signal-to-noise problems
- full chiral expansion known to $O(m_\pi^4)$ (NNLO)

Comparison of Lattice Actions

Fermion action	Doublers?	Good chiral properties?	Computational cost?
Naive	Yes (16)	Yes	Cheap
Wilson-Clover	No	No	Cheap
Staggered	Yes (4)	Yes	Cheap
Domain-Wall	No	Yes	Expensive
Overlap	No	Yes	Expensive

Our action:

- sea quarks: staggered
- valence quarks: domain-wall
- $N_f = 2 + 1 + 1$
- $\mathcal{O}(a^2)$ discretization errors

F_K/F_π Models

Goal: Determine LECs $\implies$ extrapolate to physical point

$$\left(\frac{F_K}{F_\pi}\right)_{\text{lattice}} = 1 + \delta\left(\frac{F_K}{F_\pi}\right)_{\text{NLO}} + \delta\left(\frac{F_K}{F_\pi}\right)_{\text{NNLO}} + \delta\left(\frac{F_K}{F_\pi}\right)_{\text{NNNLO}} + \delta\left(\frac{F_K}{F_\pi}\right)_{\text{FV}} + \delta\left(\frac{F_K}{F_\pi}\right)_{\mu} + \delta\left(\frac{F_K}{F_\pi}\right)_{\Lambda_\chi} + \delta\left(\frac{F_K}{F_\pi}\right)_{\alpha_S}$$

Model choices:

1. at NLO: ratio or taylor-expand
2. at NNLO: full χPT or counterterms only
3. $\mu^2 = \Lambda_\chi^2 = 4\pi\left\{F_\pi^2, F_K^2, F_\pi F_K\right\}$
4. include α_S term or not

Input:

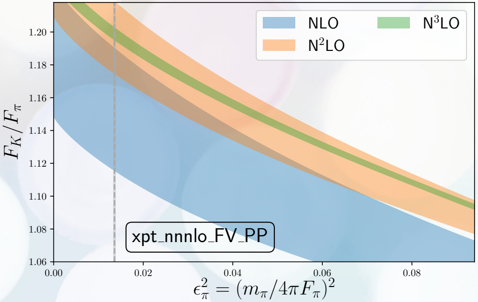
- F_K and F_π
- m_K, m_π, and m_η
- mixed meson masses
- lattice spacing

Model Parameters

order	N_{L_i}	N_χ	N_a
NLO	1	0	0
NNLO	7	2	2
NNNLO	0	3	3
Total	8	5	5

Many LECs

- constrain with priors
- check priors using empirical Bayes



Model Averaging

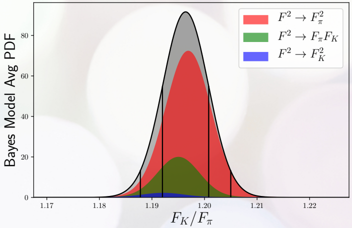
Total number of fits:

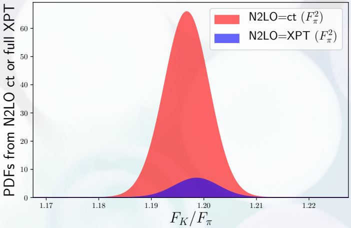
$$\underbrace{2}_{\substack{\text{expanded}\\\text{or ratio}}} \times \underbrace{2}_{\substack{\text{full } \chi\text{pt}\\\text{or ct-only}}} \times \underbrace{3}_{\substack{\text{cutoff}\\\text{choice}}} \times \underbrace{2}_{\substack{\text{incl. or}\\\text{excl. } \alpha_S}} = 24$$

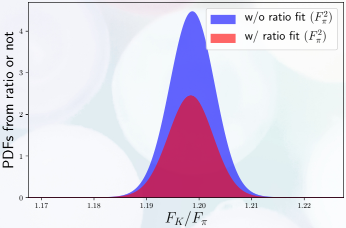
Weigh fits with Bayes factor

- marginalization $\Longrightarrow$ can compare models with different parameters
- automatically penalizes overcomplicated models

Comparison of Models





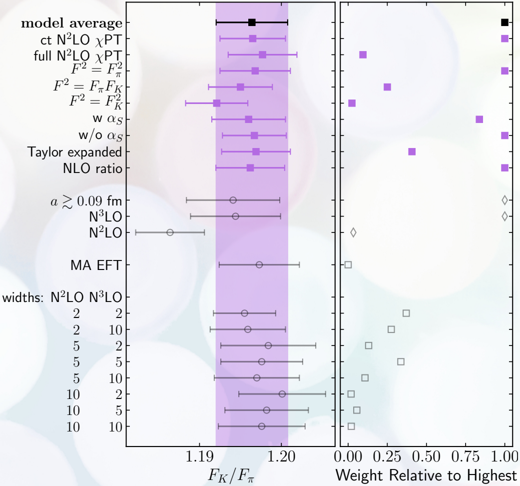


$$F_K^\pm / F_\pi^\pm = 1.1942(45)$$

Error Budget

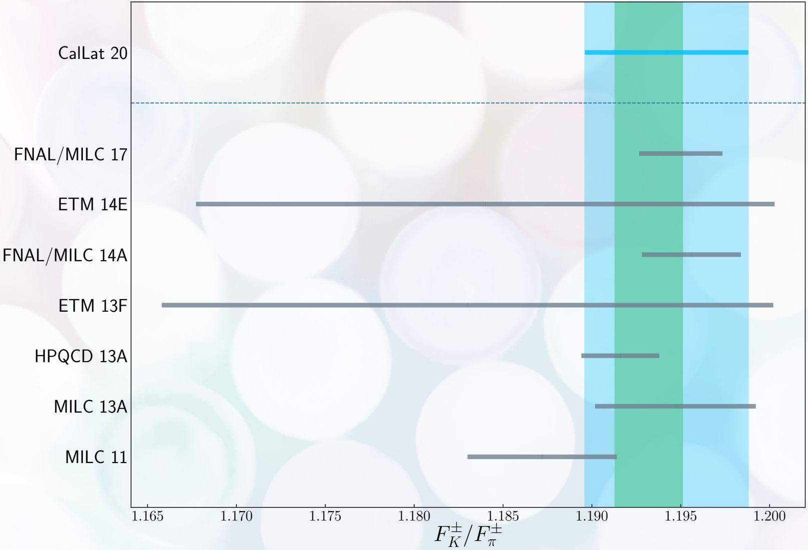
F_K/F_π	$= 1.1964 \pm 0.0044$
Statistical	0.0032
Disc	0.0020
Phys Point	0.0015
Model Unc	0.0015
Chiral	0.0012
Volume	0.0001

$$\delta\left(\frac{F_K}{F_\pi}\right)_{SU(2)} = -0.00215 \pm 0.00072$$



$$F_K^\pm/F_\pi^\pm = 1.1942(45)$$

Previous Results



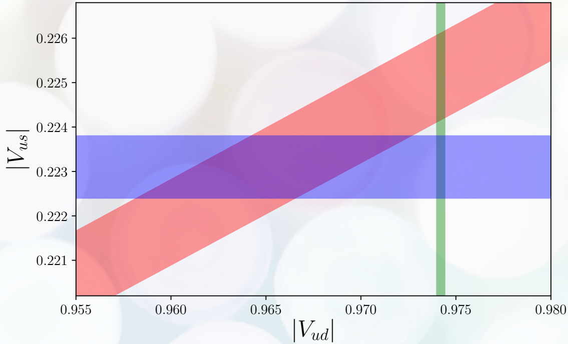
$|V_{us}|$ from F_K/F_π

This result:

$$\frac{|V_{us}|}{|V_{ud}|} = 0.2311(10)$$

With experiment:

$$|V_{us}| = 0.2251(10)$$



$$\sum_{q\in\{d,s,b\}} |V_{uq}|^2 = 0.99977(59)$$

Summary

In conclusion:

- $F_K/F_\pi \implies |V_{us}|$
- F_K/F_π is a *gold-plated* quantity and can be used to compare lattice QCD actions
- model averaging allows us to evaluate the fitness of many models without commiting to a single one

$$\boxed{F_K^\pm/F_\pi^\pm = 1.1942(45)} \implies \boxed{\sum_{q\in\{d,s,b\}} |V_{uq}|^2 = 0.99977(59)}$$

Backup slides

Model Averaging Procedure (1/2)

$$P(Y|D) = \sum_k P(Y|M_k, D) P(M_k|D)$$

with model weights

$$P(M_K|D) = \frac{P(D|M_k)P(M_k)}{\sum_l P(D|M_l)P(M_l)}$$

where $P(D|M_k)$ is obtained by marginalization

$$P(D|M_k) = \int \prod_j \mathrm{d}\theta_j^{(k)}\, P(D|\theta_j^{(k)}, M_k) P(\theta_j^{(k)}|M_k)$$

Model Averaging Procedure (2/2)

$$\mathrm{E}[Y] = \sum_k \mathrm{E}[Y|M_k]\, P(M_k|D)$$

$$\mathrm{Var}[Y] = \overbrace{\left[\sum_k \mathrm{Var}[Y|M_k] P(M_k|D)\right]}^{\text{model-averaged variance}} + \underbrace{\left[\left(\sum_k \mathrm{E}^2[Y|M_k]\, P(M_k|D)\right) - \mathrm{E}^2[Y|D]\right]}_{(\text{model uncertainty})^2}$$

Empirical Bayes Method

Let $M = \{\Pi, f\}$ denote a model. Per Bayes's theorem:

$$p(\Pi|D, f) = \frac{p(D|\Pi, f)p(\Pi|f)}{p(D|f)}$$

Assuming a uniform distribution for $p(\Pi|f)$:

$$\text{peak of } p(D|\Pi, f) \implies \text{peak of } p(\Pi|D, f)$$

where $p(D|\Pi, f)$ is the (readily available) Bayes factor

- Caveat: the uniformity assumption breaks down if we vary too many parameters separately or make our priors too narrow.

Isospin Correction

$$\delta_{SU(2)} = \frac{F_K^{\pm}}{F_\pi^{\pm}} - \frac{F_K}{F_\pi}$$

Per FLAG:

$$\delta_{SU(2)} \approx \sqrt{3}\epsilon_{SU(2)} \left[-\frac{4}{3}\left(F_K/F_\pi - 1\right) + \frac{4}{3(4\pi)^2 F_0^2}\left(m_K^2 - m_\pi^2 - m_\pi^2 \log \frac{m_K^2}{m_\pi^2} \right) \right]$$

Our paper:

$$\delta_{SU(2)} \approx \left(F_K^{\pm} - F_\pi^{\pm}\right)_{NLO} - \left(F_K - F_\pi\right)_{NLO}$$

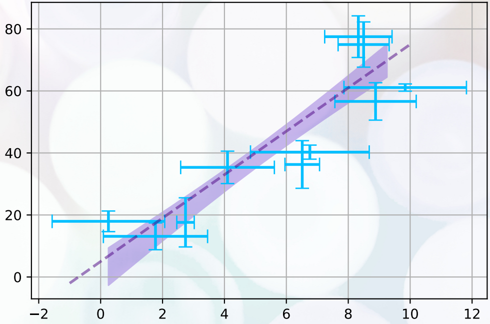
Example Model Corrections: NLO χPT w/ Taylor-Expanded Ratio

$$\delta\left(\frac{F_K}{F_\pi}\right)_{\text{NLO}} + \delta\left(\frac{F_K}{F_\pi}\right)_{\text{FV}} = \frac{5}{8}\frac{\mathcal{I}(m_\pi)}{F^2} - \frac{1}{4}\frac{\mathcal{I}(m_K)}{F^2} - \frac{3}{8}\frac{\mathcal{I}(m_\eta)}{F^2} + 4\underbrace{\frac{m_K^2 - m_\pi^2}{F^2}}_{\text{SU(3) flavor}}\underbrace{L_5}_{\text{LEC}}$$

where

$$\mathcal{I}(m) = \frac{m^2}{(4\pi)^2}\ln\left(\frac{m^2}{\mu^2}\right) + \frac{m^2}{4\pi^2}\underbrace{\sum_{|\mathbf{n}|\neq 0}\frac{c_n}{mL|\mathbf{n}|}K_1(mL|\mathbf{n}|)}_{\text{finite volume}}$$

Software: `lsqfit`/`gvar`



- Least-squares, Bayesian fitter (must specify priors)
- Tracks correlations between variables
- Allows errors-in-variables models